<table>
<tr><td colspan="6">數學系課程核心教材內容</td></tr>
<tr><td colspan="3" rowspan="2">課程名稱: (中文) 機器學習
(英文)　Machine Learning</td><td>開課單位</td><td colspan="2">應用數學所、
跨領域科學國際博士
學位學程</td></tr>
<tr><td>課程代碼</td><td colspan="2">2708073</td></tr>
<tr><td>學分數</td><td>3</td><td>必/選修</td><td>選</td><td>開課年級</td><td>一</td></tr>
<tr><td colspan="6">**Primary Objective**
This course establishes a rigorous mathematical foundation for neural networks and explores their modern applications in scientific computing and generative modeling. Students will begin from approximation theory to understand the expressive power of neural networks, then advance through optimization methods, stochastic dynamical systems, and state-of-the-art data-driven and generative modeling techniques.

在本課程中，我們將從逼近理論出發，建立類神經網路的數學理論基礎，並深入探討其在科學計算與生成模型中的現代應用。學生將理解神經網路的表達能力，進而掌握最佳化方法、隨機動力系統，以及物理與數據驅動模型的前沿技術，例如 SVD/POD 與 Koopman 分析、逼近理論、梯度流與隨機最佳化，以及擴散模型。

Grading method: Homework / Assignments (40%), Midterm Report (25%), Final Project Report (25%), In-class Paper Presentation (10%)</td></tr>
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建議參考書目	本課程沒有指定用書，教材皆以論文或者基礎數學課本章節。

課程大綱

單元主題	內容綱要	上課週數
Introduction to Machine Learning — SVD aspect	1.Singular Value Decomposition (SVD): geometry and low-rank approximation 2.Proper Orthogonal Decomposition (POD) and model reduction 3.Hilbert–Schmidt operators and spectral theory 4.Koopman operator and Dynamic Mode Decomposition (DMD) 5.Assignment 1	4
Introduction to Neural Networks — approximation aspect	1.Barron space and function class analysis 2.Universal Approximation Theorem (Hornik version) 3.Hahn–Banach Theorem and its role in approximation 4.Approximation complexity and expressive power 5.Assignment 2	4

Introduction to Neural Networks — optimization	1.First-order optimality conditions 2.Calculus of variations (first variation) 3.Gradient flow and its discretization 4.Gradient descent and convergence analysis 5.Stochastic Gradient Descent (SGD) and modern variants 6.Modified (equivalent) differential equations 7.Momentum methods: Heavy-ball and Nesterov acceleration 8.Assignment 3	6
Introduction to Neural Networks — diffusion models	1.Stochastic Differential Equations (SDEs) 2.Anderson's reverse-time SDE theorem 3.Flow matching methods 4.Diffusion models for distribution estimation 5.(Optional) Assignment 4	4

*：optional topics: Boundary integral equations, Monte-Carlo Methods (Multi-Level Monte-Carlo method)

Students may select any paper or chapter from Kutz (2013), "Data-Driven Modeling & Scientific Computation: Methods for Complex Systems & Big Data", for an in-class presentation.

Presenters will earn extra credit, but the chosen topic must be approved by the instructor beforehand.

Master's students may choose one of the following themes for their midterm and/or final report. Topics beyond this list are welcome with prior approval from the instructor.

1. Low-rank and operator methods: SVD/POD-based model reduction, Hilbert–Schmidt operator analysis, or Koopman/DMD for a dynamical system of the student's choice.
2. Approximation theory of neural networks: a study of Barron spaces, the Universal Approximation Theorem, or approximation-complexity results, with a worked example or numerical illustration.
3. Optimization dynamics: analysis of gradient flow, (stochastic) gradient descent, modified-equation analysis, or momentum methods, including a convergence experiment.
4. Diffusion and generative models: score-based / SDE formulations, Anderson's reverse-time SDE, flow matching, or a small diffusion model for distribution estimation.
5. A survey or reproduction of a recent paper connecting numerical analysis (integrators, stability, Monte-Carlo / multilevel methods, boundary integral equations) to machine learning or generative modeling.