數學系課程核心教材內容

<table>
<tr><td colspan="4" rowspan="2">課程名稱:(中文) 深度學習
(英文) Deep Learning</td><td>開課單位</td><td>應用數學碩士班</td></tr>
<tr><td>課程代碼</td><td></td></tr>
<tr><td>學分數</td><td>3</td><td>必/選修</td><td>選</td><td>開課年級</td><td>一</td></tr>
<tr><td colspan="6">Primary Objective
In this course, we will cover some of the fundamental elements in deep learning and some topics related to using Deep Learning to advance Scientific Computings. This course will review some topics in numerical analysis, linear algebra, and differential equations, and move forward to introduce the latest architecture in deep learning, for instance, the PINNs, Neural ODE, Fourier Neural Operator, and ONet.

在本課程中，我們將涵蓋深度學習的一些基本要素，以及一些與利用深度學習來推進科學計算相關的主題。本課程將回顧數值分析、線性代數和微分方程的一些主題，並進一步介紹深度學習的最新架構，例如 PINN、神經微分方程 (ODE)、傅立葉神經算子和 ONet。</td></tr>
<tr><td colspan="6">Grading method: Homework (40%), Midterm or Report (30%), Report (30%)</td></tr>
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建議參考書目	本課程沒有指定用書, 教材皆以論文或者基礎數學課本章節。 This course does not have a designated textbook; all teaching materials are papers or chapters from basic mathematics textbooks. If you really need a textbook to follow, you may find Kutz, J. N. (2013). Data-driven modeling & scientific computation: methods for complex systems & big data. OUP Oxford. as your reference. Reference papers and chapters Part 1 1. Bottou, L., Curtis, F. E., & Nocedal, J. (2018). Optimization methods for large-scale machine learning. SIAM review, 60(2), 223-311. 2. Li, Q., & Tai, C. (2019). Stochastic modified equations and dynamics of stochastic gradient algorithms I: Mathematical foundations. Journal of Machine Learning Research, 20(40), 1-47 3. Li, Q., & Tai, C. (2017, July). Stochastic modified equations and adaptive stochastic gradient algorithms. In International Conference on Machine Learning (pp. 2101-2110). PMLR. 4. Hornik, K., Stinchcombe, M., & White, H. (1989). Multilayer feedforward networks are universal approximators. Neural networks, 2(5), 359-366 5. Hornik, K. (1991). Approximation capabilities of multilayer feedforward networks. Neural networks, 4(2), 251-257. 6. Albergo, M. S., Boffi, N. M., & Vanden-Eijnden, E. (2023). Stochastic interpolants: A unifying framework for flows and diffusions. arXiv preprint arXiv:2303.08797. 7. Part 2 1. Finite Difference Methods for Ordinary and Partial Differential Equations: Steady-State and Time-Dependent Problems Ch5.3-Ch5.9 Chh6 2. Dacorogna, B. (2024). Introduction to the Calculus of Variations. World Scientific. Ch2-Ch3 Part 3 1. Cuomo, S., Di Cola, V. S., Giampaolo, F., Rozza, G., Raissi, M., & Piccialli, F. (2022). Scientific machine learning through physics–informed neural networks: Where we are and what's next. Journal of Scientific Computing, 92(3), 88. 2. Raissi, M., Perdikaris, P., & Karniadakis, G. E. (2019). Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. Journal of Computational physics, 378, 686-707. 3. Esteve-Yagüe, C., Tsai, R., & Massucco, A. (2025). Finite-difference least square methods for solving Hamilton-Jacobi equations using neural networks. Journal of Computational Physics, 524, 113721. Part 4 1. Boyce, W. E., DiPrima, R. C., & Meade, D. B. (2021). Elementary differential equations and boundary value problems. John Wiley & Sons. Ch9 2. Hairer, E., Hochbruck, M., Iserles, A., & Lubich, C. (2006). Geometric numerical integration. Oberwolfach Reports, 3(1), 805-882. Ch1 Ch2 Part 5 1. Brunton, S. L., & Kutz, J. N. (2022). Data-driven science and engineering: Machine learning, dynamical systems, and control. Cambridge University Press. Ch7 2. Chen, R. T., Rubanova, Y., Bettencourt, J., & Duvenaud, D. K. (2018). Neural ordinary differential equations. Advances in neural information processing systems, 31. Part 6 1. Li, Z., Kovachki, N., Azizzadenesheli, K., Liu, B., Bhattacharya, K., Stuart, A., & Anandkumar, A. (2020). Fourier neural operator for parametric partial differential equations. arXiv preprint arXiv:2010.08895. 2. Lu, L., Jin, P., & Karniadakis, G. E. (2019). Deeponet: Learning nonlinear operators for identifying differential equations based on the universal approximation theorem of operators. arXiv preprint arXiv:1910.03193.

課程大綱

單元主題	內容綱要	上課週數
Foundation of Deep Learning and Optimization	1. Introduction to Neural Networks. 2. Universal approximation theorem 3. Stochastic gradient descent 4. Assignment 1	3-4
Review of Numerical Methods and Calculus of Variations	1. Solving ODEs by numerical methods and stability analysis 2. Review the Calculus of Variations 3. Assignment 2	3
PINNs	1. Introduction to PINNs 2. Integrate finite difference with PINNs and error analysis 3. Assignment 3	2-3
Dynamic system	1. Introduction to dynamic systems 2. Assignment 4	2-3
Dynamic discovery problems	1. Introduction to SINDy algorithm 2. Introduction to Neural ODE and error analysis 3. Assignment 5	2-3
Deep operator learning	1. Fourier Neural Operator 2. DeepONet 3. (Optional) Assignment 6	2-3

*：optional topics : Boundary integral equations, Monte-Carlo Methods (Multi-Level Monte-Carlo method)

Students may select any paper or chapter from Kutz (2013), “Data-Driven Modeling & Scientific Computation: Methods for Complex Systems & Big Data”, for an in-class presentation.
Presenters will earn extra credit, but the chosen topic must be approved by the instructor beforehand.